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THE CARTESIAN OVAL AND RELATED CURVES AS SECTIONS OF THE ANCHOR RING.

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Let r be the radius of the circle that generates the anchor ring, and a the distance of the centre of the circle from the axis of rotation. The usual equation of the anchor ring is

$$x^2 + y^2 + z^2 - a^2 - r^2 = \pm 2a \sqrt{r^2 - z^2}.$$

This may be transposed into a form more convenient for our purpose,

$$x^2 + y^2 + z^2 + a^2 - r^2 = \pm 2a \sqrt{x^2 + y^2}. \tag{1}$$

Let $$z^2 = Px + Rx^2$$

be the general equation of a conic in the plane XZ, with the vertex at the origin, and the principal diameter in coincidence with the axis X.

When the vertex is removed to the point $-(a + r)$, i. e. to the left hand extremity of the diameter of the anchor ring, this equation becomes

$$z^2 = Rx^2 + [P + 2(a + r)R]x + (a + r)^2 R + (a + r)P, \tag{2}$$

which is also the equation of a cylinder perpendicular to the plane XZ and having the conic as its directrix.

By combining (1) and (2) we obtain

$$(1 + R)x^2 + y^2 + [P + 2(a + r)R]\, x + (a + r)^2 R + (a + r)P + a^2 - r^2 = \pm 2a \sqrt{x^2 + y^2} \tag{3}$$

as the general equation of the projection, on the plane XY, of the curve of intersection of the cylinder and anchor ring.

If $R = 0$, the conic is a parabola, and (3) becomes

$$x^2 + y^2 + Px + a^2 - r^2 + (a + r)P = \pm 2a \sqrt{x^2 + y^2}, \tag{4}$$

which will represent the *Cartesian Oval*, if P be so taken that the parabola intersects the right hand projection of the generating circle, in two given points, A and B. It will give the *Limaçon*, if P be so taken that the parabola is tangent to the circle at some point C. And we will have the *Cardioid*, when the distance between the centres is such that the point of tangency coincides with the point of intersection of the two projections of the generating circle.

Comparing (4) with the equation of the Cartesian Oval,

$$x^2 + y^2 + 2\, \frac{c}{\chi^2 - 1}\, x + \frac{d^2 - c^2}{\chi^2 - 1} = \pm 2\, \frac{\varkappa d}{\chi^2 - 1} \sqrt{x^2 + y^2}, \tag{5}$$

we see that one reduces to the other when the arbitrary constants are so taken that

$$P = 2\frac{c}{x^2 - 1}, \quad a = \frac{dx}{x^2 - 1}, \quad r = \frac{c + d + cx}{x^2 - 1}. \tag{6}$$

In both (4) and (5) the origin is taken at the focus nearest the centre, the axis of the anchor ring passing through this focus.

The Cartesian Oval passes into the Limaçon by making $d = cx$. Then we have

$$P = 2\frac{c}{x^2 - 1}, \quad a = \frac{cx^2}{x^2 - 1}, \quad r = \frac{c + 2\,cx}{x^2 - 1}. \tag{7}$$

Let the Limaçon be defined as the locus derived by drawing a secant through a fixed point on a circle whose diameter is D and laying off equal distances b each way from where the secant intersects the circle. Then the equation is

$$\rho + D\cos\varphi = \pm\, b\,,$$

and reducing (5) to that form, and combining with (7), we obtain

$$P = \frac{b^2}{D}, \quad a = \tfrac{1}{2}D\,, \quad r = \frac{b^2}{2D} + b\,. \tag{8}$$

These become the conditions for the cardioid when $b = D$.

If, now, we form the equations of the generating circle and the parabola, using the data of (7), or (8), or the values derived from (8) by making $b = D$, and find the points of intersection of the two curves; we obtain in the first case two real values, in the second case a point of tangency, and for the third this point is also the point of intersection of the two circles.

In (2), if the cylinder becomes two inclined planes, as the limiting case of an hyperbola, we have $P = 0$ and $R = \tan^2\alpha$, where α is the angle of inclination of the planes to axis X; and substituting in (3), we have the equations of three curves that bear striking resemblances to the Cartesian Oval, Limaçon, and Cardioid, according as the intersecting plane follows the corresponding conditions laid down for the intersecting cylinder.

By transforming the system of coordinates to the intersecting plane the equations of the new curves are obtained as

$$\begin{aligned}
&\rho^2 + 2\,(A\cos\varphi \pm a\sqrt{1 - \sin^2\alpha\cos^2\varphi})\,\rho + A' = 0\,,\\
&\rho\ + 2\,B\cos\varphi \pm 2\frac{r}{r + 2a}\sqrt{a - B'\cos^2\varphi} = 0\,,\\
&\rho\ + 2\,C\cos\varphi \pm 2\,C\sqrt{2 - \cos^2\varphi} = 0\,.
\end{aligned} \tag{9}$$

It will be noticed that these differ from the Cartesian Oval, Limaçon, and Cardioid only in the term involving $\cos^2\varphi$ under the radical.